クンマーの合同式とゼータ関数の左側

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1　ゼータ関数の特殊値

$$\zeta(1-r) = -\frac{B_r}{r} \tag{1}$$

1.1　$r = 2, 4, \ldots, 32$ のとき

$$\zeta(-1) = -\frac{1}{12} = -\frac{1}{2^2 \cdot 3} \tag{2}$$

$$\zeta(-3) = \frac{1}{120} = \frac{1}{2^3 \cdot 3 \cdot 5} \tag{3}$$

$$\zeta(-5) = -\frac{1}{252} = -\frac{1}{2^2 \cdot 3^2 \cdot 7} \tag{4}$$

$$\zeta(-7) = \frac{1}{240} = \frac{1}{2^4 \cdot 3 \cdot 5} \tag{5}$$

$$\zeta(-9) = -\frac{1}{132} = -\frac{1}{2^2 \cdot 3 \cdot 11} \tag{6}$$

$$\zeta(-11) = \frac{691}{32760} = \frac{691}{2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13} \tag{7}$$

$$\zeta(-13) = -\frac{1}{12} = -\frac{1}{2^2 \cdot 3} \tag{8}$$

$$\zeta(-15) = \frac{3617}{8160} = \frac{3617}{2^5 \cdot 3 \cdot 5 \cdot 17} \tag{9}$$

$$\zeta(-17) = -\frac{43867}{14364} = -\frac{43867}{2^2 \cdot 3^3 \cdot 7 \cdot 19} \tag{10}$$

$$\zeta(-19) = \frac{174611}{6600} = \frac{283 \cdot 617}{2^3 \cdot 3 \cdot 5^2 \cdot 11} \tag{11}$$

$$\zeta(-21) = -\frac{77683}{273} = -\frac{131 \cdot 593}{2^2 \cdot 3 \cdot 23} \tag{12}$$

$$\zeta(-23) = \frac{236364091}{65520} = \frac{103 \cdot 2294797}{2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13} \tag{13}$$

$$\zeta(-25) = -\frac{657931}{12} = -\frac{657931}{2^2 \cdot 3} \tag{14}$$

$$\zeta(-27) = \frac{3392780147}{3480} = \frac{9349 \cdot 362903}{2^3 \cdot 3 \cdot 5 \cdot 29} \tag{15}$$

$$\zeta(-29) = -\frac{1723168255201}{85932} = -\frac{1721 \cdot 1001259881}{2^2 \cdot 3^2 \cdot 7 \cdot 11 \cdot 31} \tag{16}$$

$$\zeta(-31) = \frac{7709321041217}{16320} = \frac{37 \cdot 683 \cdot 305065927}{2^6 \cdot 3 \cdot 5 \cdot 17} \tag{17}$$

2 円分体の相対類数

$$\#\mathrm{Cl}(\mathbb{Q}(\zeta_p)) := h_p^- h_p^+ \tag{18}$$

2.1 $p = 7, 11, 13, \ldots, 199$ のとき

$$h_7^- = 1 \tag{19}$$

$$h_{11}^- = 1 \tag{20}$$

$$h_{13}^- = 1 \tag{21}$$

$$h_{19}^- = 1 \tag{22}$$

$$h_{23}^- = 3 \tag{23}$$

$$h_{29}^- = 8 \tag{24}$$

$$h_{31}^- = 9 \tag{25}$$

$$h_{37}^- = 37 = \mathbf{37}^1 \cdot 1 \tag{26}$$

$$h_{41}^{-} = 121 \tag{27}$$

$$h_{43}^{-} = 211 \tag{28}$$

$$h_{47}^{-} = 695 \tag{29}$$

$$h_{53}^{-} = 4889 \tag{30}$$

$$h_{59}^{-} = 41241 = \mathbf{59}^{1} \cdot 699 \tag{31}$$

$$h_{61}^{-} = 76301 \tag{32}$$

$$h_{67}^{-} = 853513 = \mathbf{67}^{1} \cdot 12739 \tag{33}$$

$$h_{71}^{-} = 3882809 \tag{34}$$

$$h_{73}^{-} = 11957417 \tag{35}$$

$$h_{79}^{-} = 100146415 \tag{36}$$

$$h_{83}^{-} = 838216959 \tag{37}$$

$$h_{89}^{-} = 13379363737 \tag{38}$$

$$h_{97}^{-} = 411322824001 \tag{39}$$

$$h_{101}^{-} = 3547404378125 = \mathbf{101}^{1} \cdot 35122815625 \tag{40}$$

$$h_{103}^{-} = 9069094643165 = \mathbf{103}^{1} \cdot 88049462555 \tag{41}$$

$$h_{107}^{-} = 63434933542623 \tag{42}$$

$$h_{109}^{-} = 161784800122409 \tag{43}$$

$$h_{113}^{-} = 1612072001362952 \tag{44}$$

$$h_{127}^{-} = 2604529186263992195 \tag{45}$$

$$h_{131}^{-} = 28496379729272136525 = \mathbf{131}^{1} \cdot 217529616253985775 \tag{46}$$

$$h_{137}^{-} = 646901570175200968153 \tag{47}$$

$$h_{139}^{-} = 1753848916484925681747 \tag{48}$$

$$h_{149}^{-} = 687887859687174720123201 = \mathbf{149}^{1} \cdot 4616697044880367249149 \tag{49}$$

$$h_{151}^{-} = 2333546653547742584439257 \tag{50}$$

$$h_{157}^{-} = 56234327700401832767069245 = \mathbf{157}^{2} \cdot 2281404020463379154005 \tag{51}$$

$$h_{163}^{-} = 27085347446920770051875131636 \tag{52}$$

$$h_{167}^{-} = 28121189830322933178315382891 \tag{53}$$

$$h_{173}^{-} = 17025462666541558475167800342 65 \tag{54}$$

$$h_{179}^{-} = 772815772120302985927569747217 45 \tag{55}$$

$$h_{181}^{-} = 21142175774998754169722550153962 5 \tag{56}$$

$$h_{191}^{-} = 16500836548722365645898761132692985 9 \tag{57}$$

$$h_{193}^{-} = 54661710591356816554565075263076704 1 \tag{58}$$

$$h_{199}^{-} = 18844055286602530802019847012721555487 \tag{59}$$

$$\begin{aligned} h_{691}^{-} = {} & 29138283829700137424620641416043594733870833202236212626882500760525185 90 \\ & 42324127468927321015816568508701196399757075001182436188991486650658054 90 \\ & 58779414203836098034950070479507006964578795148218028327834768971722025 \\ = {} & \mathbf{691}^{2} \cdot 61025012156923809375913683300578650739758929051074728893678493511836 \\ & 46240213204870328396753291053853633991643641465838456031469084455436984 05 \\ & 81193929767706367030516470707882574399833636477252808421755688361985025 \end{aligned} \tag{60}$$